PROBLEM.

42. Proposed by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering, A. and M. College, College Station, Texas.

Show that the volume included between the surface represented by the equation $z=e^{-(x^2+y^2)}$ and the xy plane equals the square of the area of the section made by the zx plane, the limits of x and y being plus and minus infinity.

MECHANICS.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

20. Proposed by CHAS. E. MYERS, Canton, Ohio.

A flexible cord of given length is suspended from two points whose co-ordinates are $(x, y,)$ and (x', y'). What must be the condition of the cord in order that it may hang in the arc of a circle?

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Taking the lowest point for origin, and the horizontal and vertical lines through it for axes of x and y, and $s=a\varphi$......(1) for the intrinsic equation to the circle.

If $\pi=$ the constant horizontal component of tension at all points of the cord, the law of mass as given by Theoretical Mechanics is

$m=\dfrac{\pi}{g}\dfrac{d^2y}{dx^2}\Big\backslash\dfrac{ds}{dx}$....(2). We have $\dfrac{dy}{dx}=\tan\varphi$, $\dfrac{dx}{ds}=\cos\varphi$, $\dfrac{ds}{d\phi}=a$, from (1);

$$\frac{d^2y}{dx^2}=\frac{1}{\cos^2\varphi}\frac{d\varphi}{dx}=\frac{1}{\cos^2\varphi}\frac{d\varphi}{ds}\frac{ds}{dx}=\frac{1}{a\cos^3\varphi}\text{......(3).}$$

Then (2) gives $m=\dfrac{\pi}{g}\dfrac{a}{a^2\cos^2\varphi}=\dfrac{\pi a}{(a-y)^2}$....(4), or the mass unit varies inversely as the square of the distance below the horizontal diameter.

Excellent solutions of this problem were also received from *G. B. M. ZERR, and F. P. MATZ.*

21. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

Show that, in the wheel and axle, when a force P, acting at the circumference of the wheel, supports a weight Q upon the axle,

$$P.(R\mp\rho\sin\varepsilon)=Q(r\pm\rho\sin\varepsilon)\pm W\rho\sin\varepsilon,$$

where R, r, and ρ are the radii of the wheel, the axle, and their common axis respectively, and ε is the limiting angle of resistance.

Solution By G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia; and F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let W=weight of wheel and axle, β=the angle between P and Q and also between P and W since Q and W are parallel. The resultant of P, Q, W due to friction is $\pm\sqrt{(Q+W)^2+P^2+2P(Q+W)\cos\beta}\times\rho\sin\varepsilon$.

$\therefore PR=Qr\pm\rho\sin\varepsilon\sqrt{(Q+W)^2+P^2+2P(Q+W)\cos\beta}$.

When $\beta=0$ this becomes $PR=Qr\pm\rho\sin\varepsilon(P+Q+W)$.

$\therefore P(R\mp\rho\sin\varepsilon)=Q(r\pm\rho\sin\varepsilon)+W\rho\sin\varepsilon$.

Also solved by *ALFRED HUME.*

22. **Proposed by DE VOLSON WOOD, C. E., Professor of Mechanical and Electrical Engineering in Stevens Institute of Technology, Hoboken, New Jersey.**

A prismatic bar having a uniform angular velocity ω and a linear velocity of v feet per second, suddenly snaps (by the disappearance of the cohesive force) into an indefinite number of equal parts; required the resultant angular velocity of each piece and the locus of the parts at the end of t seconds after rupture.

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi.

Take O, the center of gravity of the bar AB, as the origin of a system of rectangular axes, the Y—axis coinciding with the direction of the motion of translation.

Let the motion of rotation be contrary to that of the hands of a clock.

Let the length of the bar be $2nl$, n being the number of equal parts into which it snaps; and let the cross-section and the density, each, be unity.

Denote the middle point of DE, any one of these equal parts, by C, any other point of DE by P.

Let $OC=R$, $OP=r$, and $\angle XOP=\theta$.

At the instant of separation P has a velocity, v, parallel to Y and a velocity, $r\omega$, perpendicular to OB.

The subsequent motion of DE may be determined by supposing it initially at rest and acted upon by such impulsive forces as are expressed in the actual motion at the instant under consideration.

The element of mass at P is acted upon by an impulsive force parallel to Y measured by the momentum $v.dr$, and by a force perpendicular to OB measured by $r\omega.dr$.

Therefore, taking moments about C, the angular velocity of DE, given by the ratio of the moment of the momentum to the moment of inertia, is

$$\frac{\int[v.\cos\theta+r\omega](r-R)dr}{\frac{2}{3}l^3}$$ the limits being $R+l$ and $R-l$.

Integrating between these limits, the numerator of this fraction becomes $\frac{2}{3}\omega l^3$.

Hence, after separation, DE will rotate about C with an angular velocity equal to that of the original bar.

C, itself, will move in the direction OY with a velocity $v+Rw.\cos\theta$ and in the direction XO with a velocity $R\omega.\sin\theta$.